

Geometry I, 2006 : Coursework 8

1. (i) [2 marks] State precisely what it means for an $n \times n$ matrix A to be *invertible*, and if A is invertible what is meant by the *inverse* of A . [Hint: consult the lecture notes.]

(ii) [3 marks] For each of the following matrices, calculate its determinant, say whether the matrix is invertible, and if it is, find its inverse:

(a) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$; (b) $\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$; (c) $\begin{pmatrix} 2 & -3 \\ 6 & -9 \end{pmatrix}$.

2. [8 marks] For each of the following statements, say whether it is true or false and justify your answer.

(i) Whenever A is an invertible $n \times n$ matrix and α is a non-zero scalar, then αA is invertible.

(ii) Whenever A and B are invertible $n \times n$ matrices, then $A + B$ is invertible.

(iii) Whenever A, B, C are $n \times n$ matrices, such that A is invertible and $AB = AC$, then we must have $B = C$.

(iv) For each invertible 2×2 matrix A , we have $\det(A) \neq 0$ and $\det(A^{-1}) = 1/\det(A)$.

3. (i) [1 mark] Using the definition of the determinant of a 2×2 matrix given in lectures, prove that $\begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix} = \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$.

(ii) [2 marks] Using the definition of the determinant of a 3×3 matrix given in lectures, prove that $\begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{vmatrix} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$.

4. [4 marks] Calculate by hand the following determinants, showing your calculations. Where appropriate, make use of Theorem 7 in the lecture notes and/or the result of the preceding exercise.

(i) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 3 & 1 \end{vmatrix}$; (ii) $\begin{vmatrix} 1 & 1 & 1234 \\ 1 & 2 & -1234 \\ 1 & 3 & 1234 \end{vmatrix}$;

(iii) $\begin{vmatrix} 27 & 1234 & 1234 \\ 19 & 1234 & 1234 \\ -15 & 1234 & 1235 \end{vmatrix}$; (iv) $\begin{vmatrix} 100 & 100 & 100 \\ -10 & -10 & -10 \\ 567 & 101 & -100 \end{vmatrix}$.

Put your name and student number on your solutions, staple them, and put them in the **Geometry I** Collecting Box in the **Basement** of the Maths Building by 5pm on Thursday 30th November.

Leonard Soicher, 23 November 2006